APPLYING SECTIONAL SEAKEEPING LOADS TO FULL SHIP STRUCTURAL MODELS USING QUADRATIC PROGRAMMING

Ming Ma[1*], Owen Hughes[2], Chengbi Zhao[3]

1. Advanced Marine Technology Center, DRS Defense Solutions, LLC ,Stevensville, MD, USA
2. Aerospace and Ocean Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA, USA
3. Naval Architecture and Ocean Engineering, South China University of Technology, Guang Zhou, P.R.China

ABSTRACT: Interest in the seakeeping loads of vessels has increased dramatically in recent years. In current design practice, methods for predicting seakeeping motions and loads are mainly in two categories, strip theory methods and 3D-panel methods. Methods based on strip theory provide reasonable motion prediction and are computationally efficient. However, many strip theory methods provide only hull girder sectional forces and moments, such as vertical bending moment and vertical shear force, which cannot be directly applied to a 3D finite element structural model. For panel based methods, the outputs include not only the global hull girder loads, but also panel pressures, which are well suited for 3D finite element analysis. However, because the panel based methods are computationally expensive, meshes used for hydrodynamic analyses are usually coarser than the mesh used for structural finite element analyses. Consequently, the panel pressure calculated from a hydrodynamic model mesh has to be transferred to the structural model mesh. The resulting discrepancy of the pressure map, regardless of what interpolation method is used, causes an imbalanced structural model. To obtain equilibrium of an imbalanced structural model, a common practice is to use the "inertia relief" approach [1]. However, this type of balancing causes a change in the hull girder load distribution, which in turn could cause inaccuracies in an extreme load analysis (ELA) and a spectral fatigue analysis (SFA). This paper presents a method to balance the structural model without using inertia relief. The method uses quadratic programming to calculate corrective nodal forces such that the resulting hull girder sectional loads match those calculated by seakeeping analyses, either by strip theory methods or 3D-panel methods. To validate the method, a 3D panel linear code, MAESTRO-Wave [2], was used to generate both panel pressures and sectional loads. A model is first loaded by a 3D-panel pressure distribution with a perfect equilibrium. The model is then loaded with only the accelerations and sectional forces and moments. The sectional forces and moments are converted to finite element nodal forces using the proposed quadratic programming method. For these two load cases, the paper compares the hull girder loads, the hull deflection and the stresses, and the accuracy proves the validity of this new method.

Keywords: Quadratic programming; Seakeeping sectional load; Finite element model load balance.

1. INTRODUCTION

With a continuous demand for innovative ship designs and more strict requirements from class societies, interest in using seakeeping loads for ship structural design has increased dramatically in recent years. Tools ranging from simple 2D strip theories to complex 3D-CFD numerical simulation methods have been used for practical designs. Most of the seakeeping tools are capable of outputting hull girder loads, such as bending moment and shear force, and some of the tools can also output panel pressure. Examples of the latter are VERES (strip theory), WAMIT, HydroStar, and Precal. Other tools such as SMP and ShipMo can only provide hull girder loads. Panel based hydrodynamic analysis is well suited for transferring hydrodynamic panel pressure to 3D finite element structural models. However, because meshes for hydrodynamic analyses are often much coarser than the corresponding finite element model meshes, it is necessary to map panel pressure from one mesh to another. Various interpolation methods have been proposed and used in design practice. However, accurately transferring seakeeping panel pressure loads to a finite element model is still a difficult task. In the hydrodynamic model solving the equations of motion and integrating the pressure gives a perfect equilibrium, but mapping the pressure to the finite element model causes imbalanced forces and moments, which must be either removed (rebalancing) or countered by a further set of correcting forces. In the "inertia relief" method these are fictitious inertial forces that come from modifying the accelerations [1] (ABS, 2010).

Recently, *Malenica et al*.(2008) [3] proposed a method which maps the panel source strength instead of the panel pressure from a hydrodynamic mesh to the structural mesh, and then formulates the equations of motion in the structural mesh. Because the equations of motion are based on the structural mesh, the method results in a perfectly balanced structural model. However, because the hydrodynamic coefficients are in general not available as an external output and because of the complexity of formulating the equations of motion, the approach is limited to the hydrodynamic tool itself. It is not practical for a third party to apply the method for mapping hydrodynamic panel pressure to a structural model. Consequently, in todays' design practice[1], the inertia relief approach is widely used to rebalance the model. However, there are two shortcomings to this approach. Firstly the additional inertial forces cause a change in the hull girder response (such as bending moment) as is shown in the next section. Secondly, the change of the accelerations has to be relatively small to ensure the fictitious inertial forces do not significantly distort the original structural response. This often requires a visual inspection and engineering judgments. For the extreme load analysis (ELA), where the number of load cases is limited, visually inspecting each load case is possible. However, for the spectral fatigue analysis (SFA), where there are thousands of load cases it is not practical.

***Corresponding author Email**: *mma@drs-ds.com* (TC-TCS012)

In this paper, an approach is proposed to achieve equilibrium based on the sectional forces and moments of the hydrodynamic analysis. The idea of applying correcting nodal forces to a finite element model is not new. CSR (2010) [4] published a guide on how to manually impose vertical forces on frames (ship sections) to obtain a shear force envelope for a partial model. The bending moment envelope is then achieved by applying moments at the ends of the partial model. This method cannot be used for a full ship model because the vertical shear force balance is just one of the six equations of equilibrium. The added nodal forces which correct the vertical shear force could cause an imbalance in the bending moment and/or torsional moment. Furthermore, it is not practical to manually adjust nodal forces to match six degrees of freedom equilibrium of a seakeeping analysis.

To solve this problem systematically, the method presented herein uses classical Quadratic Programming (QP) to find a set of corrective nodal forces for the finite element model. The mathematical objective is to minimize these corrective nodal forces, and the constraints are that the six hull girder sectional forces, must match the target values from the hydrodynamic analysis at each station (or section). The method is easy to implement and is applicable to both strip theory based methods and 3D-panel methods.

2. SIGNIFICANCE OF EQUILIBRIUM FOR FINITE ELEMENT ANALYSIS

For a floating structure, it is vital to obtain equilibrium before performing a finite element analysis. An imbalanced model causes an unrealistic result. To illustrate this, a design exercise of a floating full ship is given in this section. A finite element model was provided by NAPA Ltd of a nominal frigate 150 meters long and displacing 4000 tons. The model was created using NAPA-Steel as a molded form structural model, as shown in Figure 1. The molded form NAPA model was given a mass distribution and was balanced using NAPA's internal hydrostatic kernel. The balanced floating condition had a draft of 3.96 m, with no heel and -0.382 degree trim.

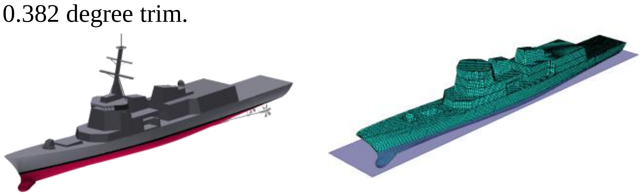


Fig.1 Finite element model generated from the NAPA molded form model

The NAPA-Steel/MAESTRO interface program can generate a full ship finite element mesh in MAESTRO format from a molded form structural model with one click of a button. The generated finite element model has over 61,000 nodes and 125,000 elements, as shown in Figure 1. This interface program also automatically translates the compartment and wetted surface definitions as MAESTRO groups. In addition, the weight distribution, tank loads and floating condition defined in NAPA hydrostatic module are also translated into MAESTRO. With a complete finite element mesh and load definition, the generated finite element model is ready for a linear static analysis without any additional manual editing.

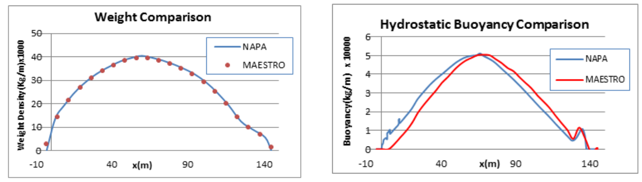
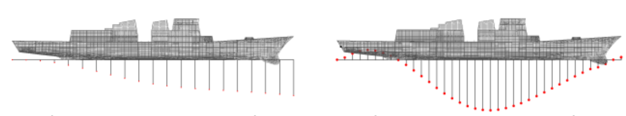


Fig.2 Weight and Buoyancy Distribution Comparison

Figures 2 shows the weight and buoyancy distribution of NAPA and MAESTRO respectively. While the weight distribution of NAPA and MAESTRO has good agreement, the buoyancy distribution between the finite element model using the initial floating position provided from NAPA and the original NAPA hydrostatic model does not. Such a discrepancy may be caused by several possible reasons: A) Rudders and propellers are usually not modeled in the finite element model for hull girder strength analysis. B) The integration schemes are different. The buoyancy calculation in NAPA is volume based, using continuous curves and/or surfaces. For MAESTRO, the buoyancy is a result of integrating the hydrostatic pressure over the faceted shell elements in the finite element model. C) There could be a user input error, which was the reason in this case. The sign of the trim angle was accidently reversed (bow down instead of bow up) such that a short aftmost portion of the ship had zero buoyancy. Figure 2 shows this clearly, but most finite element programs are not ship-specific and do not provide a buoyancy plot. We have deliberately kept this error in order to demonstrate the importance of equilibrium for a free-floating structure.

To solve the above finite element model, three nodal constraints were placed near the longitudinal neutral axis of the model to prevent the rigid body motion, with two located at the stern and one at the bow. If the model is properly balanced the restraining forces will be negligible. In order to check the balance, MAESTRO computes and plots hull girder responses ***before*** the restraints are applied. Figure 3(a) shows the vertical bending moment distribution. Since the curves are not closed, it reveals that the model is not balanced. Figure 3(b) shows the resulting bending moment distribution, which includes the reaction forces due to the constraints. Figure 4 shows the deflection and stress distribution. The lack of buoyancy at the stern and the resulting non-negligible restraint forces cause an incorrect change of sign in the bending moment and in the curvature of the deflected hull.



(a)before applying restraining forces (b)after applying restraining forces

Fig.3 Bending moment distribution

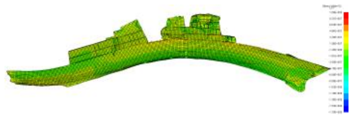


Fig. 4 Deflection and stress distribution

Finite element tools specifically developed for floating structures, such as MAESTRO, usually provide a "hydrostatic balancing kernel" by which an imbalanced finite element model can be automatically rebalanced by adjusting draft, heel and trim. After this hydrostatic balancing the MAESTRO model has a draft of 4.07m and a trim of 0.437 degrees. Once the model is balanced, the buoyancy discrepancy between NAPA and MAESTRO due to the input error has been corrected, as shown in Figure 5. Likewise the distributions of the vertical bending moment is correctly closed at both ends, and the corresponding deflection and stress distribution are as expected, as shown in Figure 6.

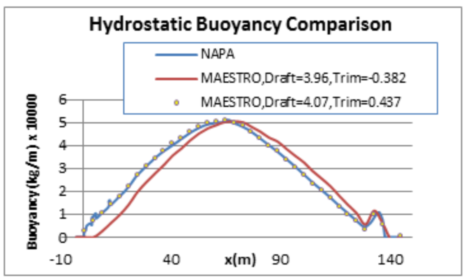


Fig.5 Buoyancy distribution comparison with MAESTRO auto-balancing

There are two situations in which hydrostatic balancing is not possible: firstly, when using general purpose finite element programs such as NASTRAN and ANSYS that do not have this feature, and secondly when the loads are derived from a linear seakeeping analysis, where the mean water surface plane is prescribed. There are two main reasons for the latter case: (1) Hydrostatic balancing can only achieve equilibrium in heave, heel and trim, but not in surge, sway and yaw. (2) A hydrostatic rebalance would cause a change of the mean water surface plane, which would require re-running the linear seakeeping analysis.

In these situations the "inertia relief" method must be used. Figure 7 shows the distributions of vertical bending moment, the corresponding deflection and stress distribution after using "inertia relief".

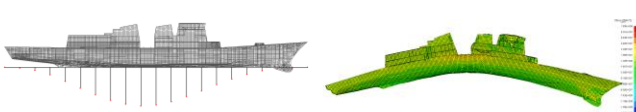


Fig. 6 Bending moment and stress distribution after hydrostatic balance

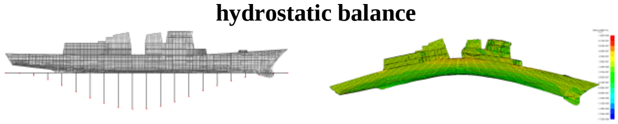


Fig. 7 Bending moment and stress distribution after inertia relief

Regardless of the method used to balance the model, hydrostatic balance or inertia relief, the hull girder responses may be changed. As shown in Figure 8, the maximum bending moment increases by 11% with hydrostatic balance and by 24% with inertia relief. In the next section a new approach is proposed to rebalance a model using Quadratic Programming (QP) which preserves all six of the hull girder forces and moments.

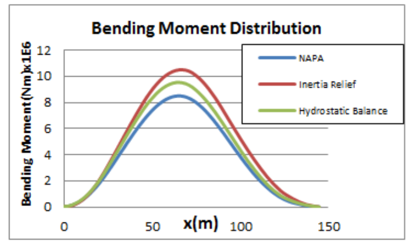


Fig. 8 Comparison of bending moment distribution

3. METHOD FOR CALCULATING CORRECTIVE NODAL FORCES

The sectional forces and moments given by a seakeeping program are a valuable description and quantification of the loads acting on the ship, and they should be used as a benchmark in generating the loads in the finite element model. If a seakeeping program gives only sectional loads, or if the pressures it provides do not give equilibrium, some corrective nodal forces are needed. We have seen that hydrostatic balancing and inertia relief both cause a departure from the sectional loads. It would be better if the corrective nodal forces are such as to preserve the known sectional loads. It is difficult to know just how to modify the nodal forces of a large finite element model to get desired resulting sectional forces and moments. But since we are seeking corrective forces, it is desirable that they be no larger than needed to achieve their purpose. Our proposal is to use quadratic programming, which is a particular type of mathematical optimization [5] (Fletcher, 1987). It minimizes a quadratic function of several variables subject to linear constraints on those variables. For the case of balancing a finite element model, the quadratic function is the magnitude of the corrective nodal forces, and the linear constraints are that the resulting sectional forces and moments must match the values from the seakeeping analysis. The problem can be set up either as a single problem for the full ship model, or as a sequence of smaller problems, each corresponding to one segment of the ship. The segments can be defined by the usual 20 stations or by any other number of cross sections, say *m*. In each case the mathematical problem is

$$\text{minimize: } q = \sum_{i=1}^{n} \left(f_{x_i}^2 + f_{y_i}^2 + f_{z_i}^2\right) \quad (1)$$

where *i* is the node index, *n* is the number of nodes in the problem (full ship or one segment), and $f_{x_i}, f_{y_i}, and\ f_{z_i}$ are the finite element nodal forces in the problem. The constraints are the *6*×*m* sectional forces and moments, as defined in equation (2).

$$
\begin{cases}
\sum_{i=1}^{n} f_{x_i} = F_{x_j} \\
\sum_{i=1}^{n} f_{y_i} = F_{y_j} \\
\sum_{i=1}^{n} f_{z_i} = F_{z_j} \\
\sum_{i=1}^{n} \left(-f_{y_i}(z_i - z_c) + f_{z_i}(y_i - y_c)\right) = M_{x_j} \\
\sum_{i=1}^{n} \left(-f_{z_i}(x_i - x_c) + f_{x_i}(z_i - z_c)\right) = M_{y_j} \\
\sum_{i=1}^{n} \left(-f_{x_i}(y_i - y_c) + f_{y_i}(x_i - x_c)\right) = M_{z_j}
\end{cases}
\quad j = 1..m \qquad (2)
$$

where j is the section index, and m is the number of the sections of the model. $m=1$ if the problem is set up for a segment. $x_c, y_c,$ and z_c are the coordinates of the center of gravity, and $x_i, y_i,$ and z_i are the finite element nodal coordinates. F_{x_j}, F_{y_j} and F_{z_j} are the cross sectional forces, and M_{x_j}, M_{y_j} and M_{z_j} are the cross sectional moments. Equations (1) and (2) can be rewritten as

$$minimize: q(\boldsymbol{X}) = \frac{1}{2}\boldsymbol{X}^{\boldsymbol{T}} G\boldsymbol{X} \qquad (3)$$

$$subject\ to: \boldsymbol{A}^T\boldsymbol{X} = \boldsymbol{b} \qquad (4)$$

where $G = 2I$, $\boldsymbol{X}$ is a vector of the finite element model's nodal forces, $\boldsymbol{A}$ is the matrix of the linear equality constraint coefficient, and $\boldsymbol{b}$ is a vector of sectional forces and moments. The quadratic programming problem can be solved by the method of *Lagrangian multipliers*. The *Lagrangian* function for this problem is

$$L(\boldsymbol{X}, \boldsymbol{\lambda}) = \frac{1}{2}\boldsymbol{X}^T\boldsymbol{G}\boldsymbol{X} - \boldsymbol{\lambda}^T(\boldsymbol{A}^T\boldsymbol{X} - \boldsymbol{b})$$

and the stationary point condition yields the equations

$$\begin{bmatrix} \boldsymbol{G} & -\boldsymbol{A} \\ -\boldsymbol{A}^T & \boldsymbol{0} \end{bmatrix} \begin{Bmatrix} \boldsymbol{X} \\ \boldsymbol{\lambda} \end{Bmatrix} = -\begin{Bmatrix} \boldsymbol{0} \\ \boldsymbol{b} \end{Bmatrix} \qquad (5)$$

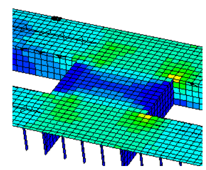
The number of variables in equation (5) is $n\times3+6\times m$. If the problem is set up for solving a segment, then m is 1, and the problem must be formulated and solved m times. Although the total number of variables is the same, the computational effort is much less. If the seakeeping program has provided the inertial forces on all the nodes and it is only the hydrodynamic pressure forces that are needed, then the QP problem need only include the wetted nodes. The basic steps of the QP method are as follows:

Table 1: The pseudo-code of the QP algorithm

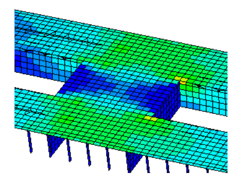
1. *Import target sectional forces and moments, along with panel pressures and nodal forces, if any, into the finite element model.*
2. *Compute sectional forces and moments based on the known loads, such as weight, panel pressure and nodal forces.*
3. *Compute the difference of the sectional forces and moments between the target and the actual.*
4. *Solve the linear equations and obtain the nodal forces of the segment. Repeat step 4 for all segments.*

4. NUMERICAL VALIDATION

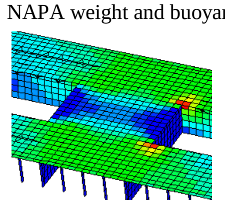
We now present two examples to compare the balancing methods and validate the QP method. In the first example we use the model described in Section 2 (with the incorrect buoyancy) to perform four types of balancing. The first two cases use the QP method with the NAPA bending moment distribution as the target. In the first case, the balancing begins with the original weight and buoyancy distribution. The corrective nodal forces are calculated such that the resulting bending moment matches the original NAPA bending moment. In the second case the QP balancing begins with *none* of the original weight and buoyancy distribution. In this case the "corrective" nodal forces are the *total* nodal forces. Again the bending moment matches the target. A static finite element analysis was performed for both cases. The hull girder deflections and longitudinal stress distributions are shown in Figures 10. A comparison of the longitudinal stress distribution in the main deck is shown in Figure 9.



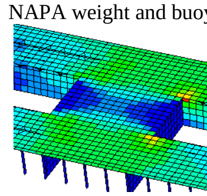
$\sigma_x = 92.1\ MPA$
QP balancing starting with the NAPA weight and buoyancy



$\sigma_x = 90.9\ MPA$
QP balancing starting without the NAPA weight and buoyancy



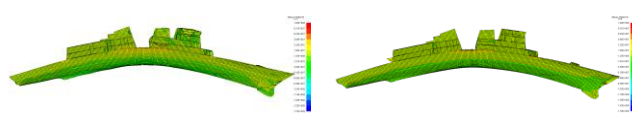
$\sigma_x = 117.9\ MPA$
Using inertia relief balancing



$\sigma_x = 104.3\ MPA$
Using hydrostatic balancing

Fig. 9 Comparison of the main deck longitudinal stress distribution

For comparison, Figure 9 includes the results from Figure 8: inertia relief and hydrostatic balancing. In all four cases the stress pattern is very similar, although the load detail of each load case is different. The peak stresses in the inertia relief case and the hydrostatic balance case are larger because they are the results of the larger corresponding hull girder bending moments, as shown in Figure 8.



with weight and buoyancy　　without weight and buoyancy

Fig. 10 Deflection and longitudinal stress

The second example is to match section loads derived from a linear seakeeping analysis. The load response of an oblique ocean wave, where longitudinal torsional moment and horizontal bending moment are not negligible, is selected to demonstrate the flexibility of the method. The model is assumed to have a forward speed of 20 knots at a heading of 135 degrees on a 10 second unit wave. A hydrodynamic analysis was performed by MAESTRO-Wave, a frequency domain potential flow linear 3D panel code. The load output of MAESTRO-Wave included both panel pressures and sectional loads. Because the equations of motion of MAESTRO-Wave are formulated based on the *structural* mesh, the pressure applied to the finite element model, along with the inertial force, result in a perfect equilibrium. The dynamic pressure distribution of the unit wave with a phase angle 0 is shown in Figure 11.

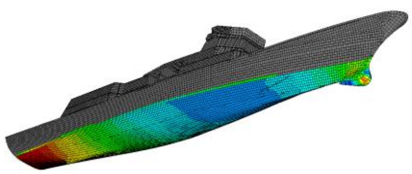


Fig. 11 Hydrodynamic Pressure Distribution

In the first load case, hydrodynamic panel pressure and the inertial forces are applied to the finite element model, and there is no need for any correction because it is already a balanced model. Then the vertical bending moment, vertical shear force, longitudinal torsional moment, horizontal bending moment and horizontal shear force are computed based on the panel pressure integration. In this load case, the individual load components, panel pressures and inertial forces are derived from first principles, so it serves as a benchmark for the following load case.

For the second load case, the inertial load was kept intact, but the hydrodynamic panel pressure was not used. The sectional loads calculated from MAESTRO-Wave were used as target values for the QP method to find the "optimal" corrective nodal forces which were equivalent to the panel pressures. Because the wave-induced panel pressures occur on the external shell, the QP method only needed to calculate corrective nodal forces at the wetted nodes of the finite element model. The sectional forces and moments were then recalculated using the inertial forces from MAESTRO-Wave and the corrective nodal forces from the quadratic programming.

For hull girder sectional loads, figures 12 to 14 compare the "exact" solution (using panel pressures with a perfect balance) and the solution without the pressures and using QP to balance the model. In all cases it is clear that the QP method achieves its purpose. The deflections and stress distributions of the full ship and the main deck, are shown in Figures 15.

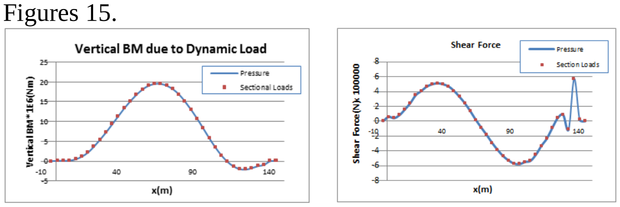


Fig. 12 Vertical bending moment and shear force

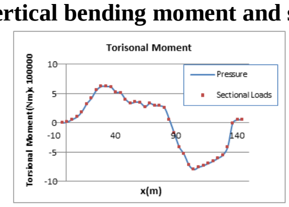


Fig. 13 Torsional moment

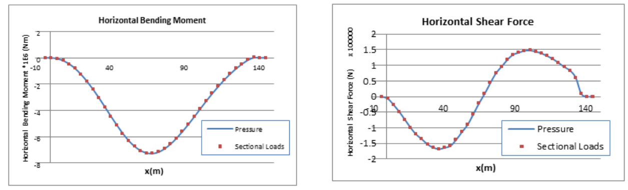
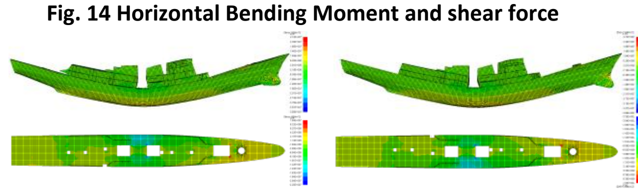


Fig. 14 Horizontal Bending Moment and shear force



Pressure loading sectional loading

Fig. 15 Stress distribution

5. CONCLUDING REMARKS

This paper presents a practical method for balancing the loads in a finite element model while matching the hull girder sectional forces and moments obtained from a seakeeping analysis. The method is flexible and easy to implement. One of the main applications for the method is to transfer seakeeping loads, obtained either by strip theory methods or 3D-panel methods, to a finite element model. Using the method, the nodal force adjustment can be applied at different levels. For example, the method can be used either with or without inertial loads, and with or without panel pressure. With the assistance of this method, the pressure mapping from a hydrodynamic mesh to a structural mesh does not have to be very accurate. The method is validated by numerical results.

6. ACKNOWLEDGMENTS

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